51. 52. 53. 54. 55. 56.

N

(9) $y'' + 10y' = 0$

$r^2 + 10r = 0$

$r(r+10) = 0$

$r = 0 \quad \vee \quad r = -10$

$y = C_1 e^{0x} + C_2 e^{-10x} = \underline{C_1 + C_2 e^{-10x}}$

(15) $y'' + 10y' + 25y = 0$

$r^2 + 10r + 25 = 0$

$(r+5)^2 = 0$

$r_1 = r_2 = -5$

$\underline{y = C_1 e^{-5x} + C_2 x e^{-5x}}$

(16) $y'' + y' + y = 0$

$r^2 + r + 1 = 0$

$r_{1,2} = \frac{-1 \pm \sqrt{-3}}{2}$

$r_{1,2} = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

$r_1 = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$y_1 = e^{-\frac{1}{2}x} \sin \frac{\sqrt{3}}{2}x$

$r_2 = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$

$y_2 = e^{-\frac{1}{2}x} \cos \frac{\sqrt{3}}{2}x$

$\underline{y_1 = C_1 e^{-\frac{1}{2}x} \sin \frac{\sqrt{3}}{2}x + C_2 e^{-\frac{1}{2}x} \cos \frac{\sqrt{3}}{2}x}$

(10) $y'' - y = 0$

$r^2 - 1 = 0$

$r^2 = 1$

$\underline{r_1 = 1} \quad \underline{r_2 = -1}$

$\underline{y = C_1 e^{x} + C_2 e^{-x}}$

(18) $y'' + 2y' + 2y = 0$

$r^2 + 2r + 2 = 0$

$r_{1,2} = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm 2i$

$\underline{y = C_1 e^{-x} \cos 2x + C_2 e^{-x} \sin 2x}$

(11) $y'' - 4y = 0$

$r^2 - 4 = 0$

$r^2 = 4$

$r_1 = 2 \quad r_2 = -2$

$\underline{y = C_1 e^{2x} + C_2 e^{-2x}}$

(19) $y'' + y = 0$

$r^2 + 1 = 0 \quad \alpha + i\beta$

$r^2 = -1 \quad r = \pm i$

$y = C_1 e^{0x} \cos x + C_2 e^{0x} \sin x$

$\underline{y = C_1 \cos x + C_2 \sin x}$

(13) $y'' + 2y' + y = 0$

$r^2 + 2r + 1 = 0$

$(r+1)^2 = 0$

$\underline{r_1 = r_2 = -1}$

$\underline{y = C_1 e^{-x} + C_2 x e^{-x}}$

(20) $y'' + 2y = 0 \quad r^2 = -2$

$r^2 + 2 = 0 \quad r = \pm i\sqrt{2}$

$\underline{y = C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x}$

(21) $y'' + 9y = 0 \quad r^2 = -9$

$r^2 + 9 = 0 \quad r = \pm 3i$

$\underline{y = C_1 \cos 3x + C_2 \sin 3x}$

(14) $y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$

$r_{1,2} = \frac{6 \pm 0}{2}$

$r_1 = r_2 = 3$

$(r-3)^2 = 0$

$\underline{y = C_1 e^{3x} + C_2 x e^{3x}}$

(23) $y''' - y'' = 0$

$r^3 - r^2 = 0$

$r^2(r-1) = 0$

$r^2 = 0 \quad \vee \quad r = 1$

$r_1 = 0 \quad r_2 = 0 \quad r_3 = 1$

$y = C_1 e^{0x} + C_2 x e^{0x} + C_3 e^{x}$

$y = C_1 + C_2 x + C_3 e^{x}$

1

$r_1=1 \quad r_2=-1 \quad r_3=-2 \qquad y_{oh}=C_1e^{x}+C_2e^{-x}+C_3e^{-2x}$

(24) $y'''+2y''-y'-2y=0$?

$r^3+2r^2-r-2=0$

$-(r^2-1)(r+2)$

$\cancel{r_1=1 \quad r_2=2 \quad r_3=1}$ 0

$\cancel{y=C_1e^{x}+C_2e^{2x}+C_3e^{x}}$

(26) $y'''-4y''-3y'+18y=0$

$r^3-4r^2-3r+18=0$

$r_1=3=r_3 \quad r_2=-2$

(27) $y'''-2y'=0$

$r^3-2r=0$

$r(r^2-2)=0$

$r_1=0 \quad (r^2=2) \quad r_2=\sqrt{2} \quad r_3=-\sqrt{2}$

$y=C_1+C_2e^{\sqrt{2}x}+C_3e^{-\sqrt{2}x}$

(28) $y'''-y=0$

$r^3-1=0$

$r^3=1$

$r_1=r_2=r_3=1$

$y=C_1e^{x}+C_2xe^{x}+C_3x^2e^{x}$

(29) $y'''+y'=0$

$r^3+r=0$

$r(r^2+1)=0 \qquad y_1=e^{0x}\cos x \qquad y_2=e^{0x}\sin x$

$r_1=0 \quad (r^2=-1) \quad r_2=i \quad r_3=-i$

$y=C_1+C_2\cos x+C_3\sin x$

(31) $y^{IV}+y'''=0$

$r^4+r^3=0$

$r^3(r+1)=0$

$r_1=r_2=r_3=0 \quad r_4=-1$

$y=C_1+C_2x+C_3x^2+C_4e^{-x}$

(32) $y^{IV}+y''=0$

$r^4+r^2=0$

$r^2(r^2+1)=0$

$r_1=r_2=0 \quad r_3=-i \quad r_4=i$

$y=C_1+C_2x+C_3\sin x+C_4\cos x$

(33) $y^{IV}+y'=0$

$r^4+r=0$

$r(r^3+1)=0 \qquad r_1=r_2=r_3=-1 \quad r_4=0$

$y=C_1e^{x}+C_2xe^{x}+C_3x^2e^{x}+C_4$

(34) $y^{IV}+y=0 \qquad r^4+1=0 \qquad r^4=-1$

$r^2=\pm1 \qquad r^2=1 \qquad r^2=-1$

$r_1=1, r_2=-1 \qquad r_3=i \quad r_4=-i$

$y=C_1e^{x}+C_2e^{-x}+C_3\cos x+C_4\sin x$

(35) $y^{IV}+2y''+y=0$

$r^4+2r^2+1=0$

$(r^2+1)^2=0$

$r^2=-1$

$r_1=r_2=i \qquad r_3=r_4=-i$

$y=C_1\cos x+C_2x\cos x+C_3\sin x+C_4x\sin x$

(37) $y^{IV}-2y''+y=0$

$r^4-2r^2+1=0$

$(r^2-1)^2=0$

$r^2=1$

$r_1=1 \quad r_2=-1 \quad r_3=1 \quad r_4=-1$

$y=C_1e^{x}+C_2e^{-x}+C_3xe^{x}+C_4xe^{-x}$

B

2

(39) $y^{IV} - 6y'' + 9y = 0$ $r^4 - 6r^2 + 9 = 0$ $(r^2-3)^2 = 0$ $r^2 = 3$

$r_1 = r_2 = \sqrt{3}$ $r_3 = r_4 = -\sqrt{3}$

$y = C_1 e^{\sqrt{3}x} + C_2 x e^{\sqrt{3}x} + C_3 e^{-\sqrt{3}x} + C_4 x e^{-\sqrt{3}x}$

(40) $y^{V} + y^{IV} = 0$ $r^5 + r^4 = 0$ $r^4(r+1) = 0$ $r_1 = r_2 = r_3 = r_4 = 0$

$r^4 = 0$ $r = -1$ $r_5 = -1$

$y = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 e^{-x}$

(41) $y^{V} + y''' = 0$ $r^5 + r^3 = 0$ $r^3(r^2+1) = 0$ $r^3 = 0$ $r^2 = -1$

$r_1 = r_2 = r_3 = 0$ $r_4 = i$ $r_5 = -i$

$y = C_1 + C_2 x + C_3 x^2 + C_4 \cos x + C_5 \sin x$

(42) $y^{V} - y''' = 0$ $r^5 - r^3 = 0$ $r^3(r^2-1) = 0$ $r^3 = 0$ $r^2 = 1$

$r_1 = r_2 = r_3 = 0$ $r_4 = 1$ $r_5 = -1$

$y = C_1 + C_2 x + C_3 x^2 + C_4 e^{x} + C_5 e^{-x}$

(43) $y^{V} - y'' = 0$ $r^5 - r^2 = 0$ $r^2(r^3 - 1) = 0$ $r^2 = 0$ $r^3 = 1$

$r_1 = r_2 = 0$ $r_3 = r_4 = r_5 = 1$

$y = C_1 + C_2 x + C_3 e^{x} + C_4 x e^{x} + C_5 x^2 e^{x}$

$r_{1,2} = \frac{-1 \pm 3}{2}$ $r_1 =$

(45) $y^{V} + y^{IV} - 2y''' = 0$ $r^5 + r^4 - 2r^3 = 0$ $r^3(r^2 + r - 2) = 0$

$r_1 = r_2 = r_3 = 0$ $r_4 = 1$ $r_5 = -2$

$y = C_1 + C_2 x + C_3 x^2 + C_4 e^{x} + C_5 e^{-2x}$

$(r-1)^2$

(46) $y^{V} - 2y''' + y' = 0$ $r^5 - 2r^3 + r = 0$ $r(r^4 - 2r^2 + 1) = 0$

$r(r^2-1)^2 = 0$

$y = C_1 + C_2 e^{x} + C_3 x e^{x} + C_4 e^{-x} + C_5 x e^{-x}$

$r^2 = 1$

$r_1 = 0$ $r_2 = r_3 = 1$ $r_4 = r_5 = -1$

(47) $y^{V} + y' = 0$ $r^5 + r = 0$ $r(r^4 + 1) = 0$ $r^4 = -1$

$r_1 = 0$ $r_2 = r_3 = i$ $r_4 = r_5 = -i$

$y = C_1 + C_2 \cos x + C_3 x \cos x + C_4 \sin x + C_5 x \sin x$

3

(49) $y^{VI} - y^{IV} = 0$ $r^6 - r^4 = 0$ $r^4(r^2 - 1) = 0$ $r^2((r^2-1)(r^2+1)) = r^2((r-1)(r+1)(r^2+1)) = 0$

~~$r_1 = r_2 = 0$~~ $r^4 = 1$ $r_3 = r_4 = 1$ $r_5 = r_6 = -1$

~~$y = C_1 + C_2 x + C_3 e^{x} + C_4 x e^{x} + C_5 e^{-x} + C_6 x e^{-x}$~~

$r_{1/2} = 0$ $r_3 = 1$ $r_4 = -1$ $r_{5/6} = \pm i$

(50) $y^{VI} + 2y^{V} + y^{IV} = 0 \quad r^6 + 2r^5 + r = 0 \quad r(r^5 + 2r^4 + 1) = 0$

5.2 Nehomogena linearna dif. j-na sa konst. koeficijentima

SVESKA

(2) $y'' - 3y' + 2y = e^{3x}(x + x^2)$

$r^2 - 3r + 2 = 0$

$r_{1,2} = \frac{3 \pm 1}{2}$

$r_1 = 2 \quad r_2 = 1$

$y = c_1 e^x + c_2 e^{2x}$

$\alpha = 3 \quad \beta = 0$

$\alpha + i\beta = 3 \notin \{r_1, r_2\}$

$S = 0$

$y_p = e^{3x}(ax^2 + bx + c)$

$y_p' = [3e^{3x}(ax^2 + bx + c)] + e^{3x}(2ax + b)$

$y_p'' = 9e^{3x}(ax^2 + bx + c) + 3e^{3x}(2ax + b) + 3e^{3x}(2ax + b) + e^{3x} \cdot 2a$

$9e^{3x}(ax^2 + bx + c) + 6e^{3x}(2ax + b) + 2ae^{3x}$

$e^{3x}(9ax^2 + 9bx + 9c + 12ax + 6b + 2a) =$

$e^{3x}(x^2(9a) + x(9b + 12a) + 2a + 6b + 9c) = e^{3x}(x - x^2)$

$9ax^2 + 3x(4a + 3b) + (2a + 6b + 9c) = x - x^2$

$y_{op} = y_{oh} + y_p = C_1 e^x + C_2 e^{2x} + e^{3x}\left(\frac{1}{2}x^2 - x + 1\right)$ – proveriti da li je dobro

$\alpha + i\beta$

$e^{\alpha x} \cos \beta x$

(*) $y'' - 2y' + y = 6xe^x$

$r^2 - 2r + 1 = 0$

$(r - 1)^2 = 0$

$r_1 = r_2 = 1$

$y = c_1 e^x + c_2 x e^x$

$\alpha = 1 \quad \beta = 0$

$\alpha + i\beta = 1 = r_1 = r_2$

$\boxed{S = 2}$

$y_p = e^x(ax + b) \cdot x^2 = e^x(ax^3 + bx^2)$

$y_p' = e^x(ax^3 + bx^2) + e^x(3ax^2 + 2bx)$

$e^x(ax^3 + 3ax^2 + bx^2 + 2bx)$

$y_p'' = e^x(ax^3 + 3ax^2 + bx^2 + 2bx) +$

$+ e^x(3ax^2 + 6ax + 2bx + 2b)$

d.j. $\cancel{e^x}(ax^3 + 6ax^2 + bx^2 + 4bx + 6ax + 2b) - 2\cancel{e^x}(ax^3 + 3ax^2 + bx^2 + 2bx) +$

$+ \cancel{e^x}(ax^3 + bx^2) = 6x\cancel{e^x}$

$ax^3 + 6ax^2 + bx^2 + 4bx + 6ax + 2b = 2ax^3 - 6ax^2 + 2bx^2 - 4bx + ax^3 + bx^2 = 6x$

$6ax + 2b = 6x$

$a = 1 \quad b = 0$

$$\boxed{y_p = e^x(x^3) = e^x \cdot x^3}$$

$\int \frac{1}{\sin x} \quad \ln\left|\operatorname{tg}\frac{x}{2}\right|$

$\int \frac{1}{\cos x} \quad -\ln\left|1 - \operatorname{tg}\frac{x}{2}\right|$

$\boxed{7}$ $y'' + y = x \sin x$

$r^2 + 1 = 0$

$r^2 = -1$

$r = \pm i$

$y_{hom} = c_1 \cos x + c_2 \sin x$

nehom $\alpha + i\beta = 0 + 1 \cdot i = i \quad i \in (r_1, r_2) \quad S = 1$

$y_p = [(ax+b)\cos x + (cx+d)\cdot \sin x] \cdot x^1$

$y_p' = [a\cos x - (ax+b)\sin x + c\sin x + (cx+d)\cos x]x + [(ax+b)\cos x + (cx+d)\sin x]$

$= [\cos x(a + cx + d) + \sin x(c - ax - b)]x + [\cos x[ax+b] + \sin x[cx+d]]$

$y_p'' = [\cos x(a+cx+d) + \sin x(c-ax-b)] + x[(c)\cos x - (a+cx+d)\sin x)] + a\sin x + (c-ax-b)\cos x$

$+ (a\cos x - (ax+b)\sin x) + c\sin x + (cx+d)\cos x$

$y_p'' = \cos x(a + cx + d + xc + c - ax - b + a + cx + d) + \sin x(c - ax - b - x - acx^2 - dx - a - ax - b + c)$

$(2a + 2d + 3cx + c - b)\cos x + (2c - 2ax - 2b - a - acx^2 - x)\sin x$

$(2a + 2d + 3cx + c - b)\cos x + (2c - 2b - a - 2ax - acx^2 - x)\sin x = x\sin x$

5.2

(1) $y'' - y = x$ $\alpha = 0$ $\beta = 0$

$r^2 - 1 = 0$ $\alpha + i\beta = 0 \notin$ $S = 0$

$r^2 = 1$

$r_1 = 1$ $r_2 = -1$

$y_{hom} = C_1 e^x + C_2 e^{-x}$

$y_p = ax + b$

$y_p' = a$

$y_p'' = 0$

$-ax + b = x$

$a = -1$ $b = 0$ $y_p = -x$

$y = C_1 e^x + C_2 e^{-x} - x$ opšte rešenje

(2) $y'' - y = e^x$ $\alpha = 1$ $\beta = 0$

hom $r^2 - 1 = 0$ $\alpha + i\beta = 1 + 0 = 1 \in$ $S = 1$

$r^2 = 1$

$r_1 = 1$ $r_2 = -1$

$y_{hom} = C_1 e^x + C_2 e^{-x}$

$y_p = ae^x \cdot x$

$y_p' = ae^x x + ae^x = ae^x(x+1)$

$y_p'' = ae^x(x+1) + ae^x = ae^x(x+2)$

dj

$ae^x(x+2) - ae^x \cdot x = e^x$

$\cancel{ax} + 2a - \cancel{ax} = 1$

$2a = 1$

$\boxed{a = \frac{1}{2}}$

$y_p = \frac{1}{2} x e^x$

$y = C_1 e^x + C_2 e^{-x} + \frac{1}{2} x e^x$

(4) $y'' + y' = x^2 + 1$ $ax^2 + bx + c$

$r^2 + r = 0$

$r(r+1) = 0$

$r_1 = 0$ $r_2 = -1$

$y_{hom} = C_1 + C_2 e^{-x}$

(5) $y'' + y' = \cos x$

$\alpha = 0 \quad \beta = 1$

hom $\quad r^2 + r = 0 \quad r(r+1) = 0$

$\alpha + i\beta = 0 + 1 \cdot i = i \notin \{r_1, r_2\}$ $\quad$ $S = 0$

$r_1 = 0 \quad r_2 = -1$

$y_{hom} = C_1 + C_2 e^{-x}$

$y_p = a\cos x + b\sin x$

$y_p' = -a\sin x + b\cos x$

$y_p'' = -a\cos x - b\sin x$

$\cancel{-a\cos x - b\sin x - a\sin x + b\cos x = \cos x}$

$-a(\cos x + \sin x) + b(\cos x - \sin x) = \cos x$

$\cos x\underbrace{(b-a)}_{1} + \sin x\underbrace{(b+a)}_{0} = \cos x$

$b - a = 1$

$b + a = 0$ +

$2b = 1$

$b = \frac{1}{2}$ $\quad a = -\frac{1}{2}$

dx

$y_p = -\frac{1}{2}\cos x + \frac{1}{2}\sin x$

$$\boxed{y = C_1 + C_2 e^{-x} + \frac{1}{2}(\sin x - \cos x)}$$

(4) $y'' + y' = x^2 + 1$

$r_1 = 0 \quad r_2 = -1$

$y_{hom} = C_1 + C_2 e^{-x}$

méthode $\quad y = C_1(x)e^{0} + C_2(x)e^{-x}$

$C_1'(x) + C_2'(x)e^{-x} = 0$

$C_1'(x) \cdot 0 + C_2'(x)(-e^{-x}) = x^2 + 1$

$\Delta = \begin{vmatrix} 1 & e^{-x} \\ 0 & -e^{-x} \end{vmatrix} = -e^{-x} - 0 = -e^{-x}$

$\Delta_1 = \begin{vmatrix} 0 & e^{-x} \\ x^2+1 & -e^{-x} \end{vmatrix} = 0 - e^{-x}(x^2+1)$

$\Delta_2 = \begin{vmatrix} 1 & 0 \\ 0 & x^2+1 \end{vmatrix} = x^2 + 1$

$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\cancel{e^{-x}}(x^2+1)}{-\cancel{e^{-x}}} = x^2 + 1$

$C_1(x) = \int (x^2+1)\,dx + D_1$

$= \int x^2\,dx + \int dx + D_1$

$C_1(x) = \frac{1}{3}x^3 + x + D_1$

$C_2'(x) = \frac{\Delta_2}{\Delta} = \frac{x^2+1}{-e^{-x}}$

$C_2(x) = -\int \frac{x^2}{e^{-x}} - \int \frac{dx}{e^{-x}} + D$

(7) $y'' + y = x^3 + x^2$

$r^2 + 1 = 0$

$r^2 = -1 \qquad r = \pm i$

$C_1'(x)\cos x + C_2'(x)\sin x = 0$

$C_1'(x)(-\sin x) + C_2'(x)\cos x = x^3 + x^2$

$y = C_1 \cdot \cos x + C_2 \cdot \sin x$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ x^3+x^2 & \cos x \end{vmatrix} = 0 - (x^3+x^2)\sin x$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & x^3+x^2 \end{vmatrix} = (\cos x)(x^3+x^2)$$

$Y = Ax^3 + Bx^2 + Cx + D = x^3 + x^2$

$Y' = 3Ax^2 + 2Bx + C$

$Y'' = 6Ax + 2B$

rešeň

$$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\sin x(x^3+x^2)}{1}$$

$$C_1(x) = -\int (x^3+x^2)\sin x\,dx + D_1$$

$$= -\int x^3 \sin x\,dx - \int x^2 \sin x\,dx$$

$6Ax + 2B + Ax^3 + Bx^2 + Cx + D = x^3 + x^2$

$u = x^3 \qquad dv = \sin x\,dx$

$du = 3x^2\,dx \qquad v = -\cos x$

$Ax^3 = x^3$

$A = 1$

$Bx^2 = x^2$

$B = 1$

$$-\left(-x^3\cos x - \int -\cos x \cdot 3x^2\,dx\right)$$

$$x^3\cos x + 3\int x^2 \cos x\,dx$$

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x$

$$3\left(x^2 \sin x - \int 2x \sin x\,dx\right)$$

$$-2\int x \sin x\,dx$$

$u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$$-x\cos x + \int \cos x\,dx$$

$$(-x\cos x + \sin x)$$

$$3\left(x^2 \sin x + 2x\cos x - 2\sin x\right)$$

$$x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x$$

$\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x \, dx$

$du = 2x\,dx \qquad v = -\cos x \, dx$

$-x^2\cos x + \int \cos x \; 2x \, dx$

$2\int x \cos x$

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \sin x$

$x \sin x - \int \sin x \, dx$

$x^3\cos x + 3x^2 \sin x + 6x\cos x - 6\sin x - x^2\cos x + 2(x\sin x + \cos x)$

$+ 2x\sin x + 2\cos x + D_1$

$C_1(x) = \cos x \left(x^3 - x^2 + 6x + 2\right) + \sin x\left(3x^2 + 2x - 6\right) + D_1$

$C_2'(x) = \frac{\Delta_2}{\Delta} = (x^3 + x^2)\cos x$

$x^2 \sin x + 2x\cos x - 2\sin x$

$C_2(x) = \int (x^3 + x^2)\cos x \, dx = \int x^3 \cos x + \int x^2 \cos x$

$u = x^3 \qquad dv = \cos x \, dx$

$du = 3x^2 dx \qquad v = \sin x$

$x^3 \sin x - 3\int x^2 \sin x \, dx = -x^2\cos x + 2x\sin x + 2\cos x + D$

$x^3 \sin x + 3x^2\cos x - 6x\sin x - 6\cos x - 3D_1 + x^2 \sin x + 2x\cos x - 2\sin x$

$C_2(x) = \sin x\left(x^3 + x^2 - 6x - 2\right) + \cos x\left(3x^2 + 2x - 6\right) + 3D_2$

$y =$

(10) $y'' + y' - 6y = xe^x$

$r^2 + r - 6 = 0$

$r_{1,2} = \frac{-1 \pm 5}{2}$

$r_1 = 2 \quad r_2 = -3$

$y_{hom} = C_1 e^{2x} + C_2 e^{-3x}$

$\alpha = 1 \quad \beta = 0$

$\alpha + i\beta = 1 + 0 \cdot i = 1 \quad \notin \quad S = 0$

$y_p = e^x(ax + b) \cdot x^0$

$y_p' = e^x(ax + b) + e^x \cdot a$

$y_p'' = e^x(ax+b) + e^x a + ae^x = e^x(ax+b) + 2e^x a$

$\cancel{e^x}(ax+b) + 2a\cancel{e^x} + a\cancel{e^x} + \cancel{e^x}(ax+b) - 6\cancel{e^x}(ax+b) = x\cancel{e^x}$

$3a - 4(ax + b) = x$

$3a - 4ax - 4b = x$

$-4ax + 3a - 4b = x$

$-4a = 1 \qquad 3a - 4b = 0$

$a = -\frac{1}{4} \qquad -\frac{3}{4} - 4b = 0$

$4b = -\frac{3}{4}$

$b = -\frac{3}{16}$

$y_p = e^x\left(-\frac{1}{4}x - \frac{3}{16}\right)$ Nevena

$y = C_1 e^{2x} + C_2 e^{-3x} + e^x\left(-\frac{1}{4}x - \frac{3}{16}\right)$

(13) $y'' + 2y' + y = e^x$ $\alpha = 1 \quad \beta = 0$

$r^2 + 2r + 1 = 0$ $\alpha + i\beta = 1 + 0i = 1 \notin \quad s = 0$

$(r+1)^2 = 0$

$r_1 = r_2 = -1$

$y = C_1 e^{-x} + C_2 x e^{-x}$

$y_p = ae^x$ $y_p = \frac{1}{4}e^x$

$y_p' = ae^x$

$y_p'' = ae^x$ $y = C_1 e^{-x} + C_2 x e^{-x} + \frac{1}{4}e^x$

d.j. $a\cancel{e^x} + 2a\cancel{e^x} + a\cancel{e^x} = \cancel{e^x}$

$4a = 1$

$a = \frac{1}{4}$

(15) $y'' - 2y' = x + \sin 2x$ $f_1(x) = x \quad \alpha = 0 \quad \beta = 0$

$r^2 - 2r = 0$ $y_{p_1} = (ax+b)x$

$r(r-2) = 0$ $f_2(x) = \sin 2x \quad \beta = 2 \quad \alpha = 0$

$r_1 = 0 \quad r_2 = 2$ $\alpha + i\beta = 0 + 2i = 2i \notin \quad s = 0$

$(a\cos 2x + b\sin 2x)$

$y_{hom} = C_1 + C_2 e^{2x}$

$-2\sin 2x$

$y_p = x(ax+b) + (a\cos 2x + b\sin 2x)$

$y_p' = [(ax+b) + xa] + [-2a\sin 2x + 2b\cos 2x]$

$y_p'' = [a + a] + [-2a \cdot 2\cos 2x + 2b(-2)\sin 2x]$

$2a - 4a\cos 2x - 4b\sin 2x$

d.j.

$2a - 4a\cos 2x - 4b\sin 2x - 2xa - 2(ax+b) + 4a\sin 2x - 4b\cos 2x = x + \sin 2x$

$x(-2a - 2a) + \cos 2x(-4a - 4b) + \sin 2x(4a - 4b) + 2a - 2b = x + \sin 2x$

1 0 1 0

$2a - 2b = 0$ $4a - 4b = 0$ $2a - 2b = 0$ $a = b$ $-\frac{1}{4} = \frac{1}{4} \cdot -\frac{1}{4}$

$-4a = 1$ $4a - 4b = 1$ $a = -\frac{1}{4}$ $-\frac{1}{4} +$

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$y'' - 2y' = x + \sin 2x$ $\quad r_1 = 0 \; r_2 = 2$ $\quad \bar{y}_{hom} = C_1 + C_2 e^{2x}$

$f_1(x) = x \quad \alpha = 0 \quad \beta = 0 \quad 0 + i \cdot 0 = 0 \quad S_1 = 1$

$y_1 = (ax + b)x$

$y_1' = ax + b + ax = 2ax + b$

$y_1'' = 2a$

$2a - 4ax - 2b = x$

$-4ax + 2a - 2b = x$

$a = -\frac{1}{4}$ $\quad b = -\frac{1}{4}$

$y_{p_1} = \left(-\frac{1}{4}x - \frac{1}{4}\right)x$

$0 + 2i - 2i \not\in$

$f_2(x) = \sin 2x \quad \alpha = 0 \quad \beta = 2$

$s = \emptyset$

$y_2 = (a\cos 2x + b\sin 2x) \cdot x$

$y_2' = (-2a\sin 2x + 2b\cos 2x) \cdot x + (a\cos 2x + b\sin$

$y_2'' = (-4a\cos 2x - 4b\sin 2x) \cdot x$

$y_2'' = (-4a\cos 2x - 4b\sin 2x) \cdot x + (-2a\sin 2x + 2b\cos 2x) + (-2a\sin 2x + 2b\cos 2x)$

$= x(-4a\cos 2x - 4b\sin 2x) + (-4a\sin 2x + 4b\cos 2x)$

$+ 2(a\cos 2x + b\sin 2x) - 2x(2b\cos 2x - 2a\sin 2x) = x + \sin 2x$

$-4a\sin 2x + 4b\cos 2x - 2a\cos 2x - 2b\sin 2x + x(+4a\sin 2x - 4b\cos 2x) = x + \sin 2x$

$+ x(-4a\cos 2x - 4b\sin 2x)$

A $\quad A$

$x(-4a\cos 2x - 4b\sin 2x + 4a\sin 2x - 4b\cos 2x)$

$\sin 2x(-4a - 2b) + \cos 2x(4b - 2a)$

$1 \qquad 0$

$4b - 2a = 0$

$-8b - 2b = 1$ $\qquad 2b - a = 0$

$-10b = 1$ $\qquad a = 2b$

$b = -\frac{1}{10}$ $\quad a = -\frac{1}{5}$

$-4 \cdot -\frac{2}{10}\cos 2x - 4\left(-\frac{1}{10}\right)\sin 2x$

$+ 4\left(-\frac{2}{10}\right)\sin 2x - 4 \cdot -\frac{1}{10}\cos 2x$

$\frac{8}{10}\cos 2x + \frac{4}{10}\cos 2x$

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(28) $y''' - y = 2e^x$ $\alpha = 1$ $\beta = 0$

$r^3 - 1 = 0$ $s = 3$

$r^3 = 1$

$r_1 = r_2 = r_3 = 1$

$y_{hom} = c_1 e^x + c_2 x e^x + c_3 x^2 e^x$

$y_p = a x^3 e^x$

$y_p' = 3ax^2 e^x + a x^3 e^x$

$y_p'' = 6axe^x + 3ax^2 e^x + 3ax^2 e^x + ax^3 e^x$

$= (6ax e^x) + (6ax^2 e^x) + (ax^3 e^x)$

$\cancel{6} axe^x (6 + \cancel{6}x + x^2)$

$y_p''' = (axe^x(6 + 6x + x^2))'$

$6ae^x + 6axe^x + 12axe^x + 6ax^3e^x + 3ax^2e^x + ax^3e^x = y_p^{IV}$

$6a\cancel{e^x} + 6ax\cancel{e^x} + 12ax\cancel{e^x} + 6ax^3\cancel{e^x} + 3ax^2\cancel{e^x} + \cancel{ax^3e^x} - \cancel{ax^3e^x} = 2\cancel{e^x}$

$6a + 6ax + 12ax + 6ax^3 + 3ax^2 = 2$

$x(6a + 12a + 6ax^2 + 3ax) + 6a = 2$

$a = \frac{1}{3}$

(32.) $y^{IV} + y'' = 1 + e^x$

$r^4 + r^2 = 0$

$r^2(r^2 + 1) = 0$

$r_1 = r_2 = 0$

$r_3 = i$ $r_4 = -i$

$y_{hom} = c_1 + c_2 x + c_3 \cos x + c_4 \sin x +$

$f(x) = 1$ $\alpha = 0$ $\beta = 0$ $s = 2$

$y_1 = ax^2$

$y_1' = 2ax$

$y_1'' = 2a$

$y_1''' = 0$

$y^{IV} = 0$

c_1 $\cancel{0} + 0 + 2a = 1$

$a = \frac{1}{2}$ $y_{p_1} = \frac{1}{2}x^2$

$f(x) = e^x$ $\alpha = 1$ $\beta = 0$ $s = 0$

$y_2 = ae^x$

$y_1' = ae^x$

$y_1'' = ae^x$

$y''' = ae^x$

$y^{IV} = ae^x$

c_2 $2a\cancel{e^x} = \cancel{e^x}$

$a = \frac{1}{2}$ $y_2 = \frac{1}{2}e^x$

Lagranž

① $y'' - y' = \frac{2-x}{x^3} e^x$

$r^2 - r = 0$

$r(r-1) = 0$

$r_1 = 0 \quad r_2 = 1$

$y_{hom} = \underline{C_1 + C_2 e^x}$ $\quad$ **neh.** $\quad y_{neh} = C_1(x) + C_2(x) e^x$

$y_1 = 1 \quad y_1' = 0 \quad C_1'(x) + C_2'(x) e^x = 0$

$y_2 = e^x \quad y_2' = e^x \quad 0 + C_2'(x) e^x = \frac{2-x}{x^3} e^x$

$\Delta = \begin{vmatrix} 1 & e^x \\ 0 & e^x \end{vmatrix} = e^x$ $\qquad \Delta_2 = \begin{vmatrix} 1 & 0 \\ 0 & \frac{2-x}{x^3} e^x \end{vmatrix} = \frac{2-x}{x^3} e^x$

$\Delta_1 = \begin{vmatrix} 0 & e^x \\ \frac{2-x}{x^3} e^x & e^x \end{vmatrix} = 0 - e^{2x} \frac{2-x}{x^3}$ $\qquad C_2' = \frac{\Delta_2}{\Delta} = \frac{2-x}{x^3}$

$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{e^{2x} \frac{2-x}{x^3}}{e^x} = e^x \frac{2-x}{x^3}$

$C_1(x) = \int e^x \frac{2-x}{x^3} dx \qquad 2\int \frac{e^x}{x^3} - \int \frac{e^x}{x^2}$

$x^{-3} e^x$

$C_2(x) = -\frac{1}{x^2} - \frac{1}{x} + D_2$

$u = e^x$

$du = e^x dx$

$dv = \frac{1}{x^3}$

$v = -\frac{1}{2x^2}$

$-\frac{e^x}{2x^2} + \frac{1}{2} \int \frac{e^x}{x^2}$

$-\frac{e^x}{x^2} + \cancel{\int \frac{e^x}{x^2}} - \cancel{\int \frac{e^x}{x^2}}$

$C_1(x) = -\frac{e^x}{x^2} + D_1$

$y = \left(-\frac{e^x}{x^2} + D_1\right) + \left(-\frac{1}{x^2} - \frac{1}{x} + D_2\right) e^x$

(3) $y'' - y = \frac{e^x}{1+e^x}$

$$r^2 - 1 = 0$$

$$r^2 = 1$$

$$r_1 = 1 \quad r_2 = -1$$

$$y_{hom} = C_1 e^x + C_2 e^{-x}$$

$$y_{neh} = C_1(x) e^x + C_2(x) e^{-x}$$

$$C_1'(x) e^x + C_2'(x) e^{-x} = 0$$

$$C_1'(x) e^x - C_2'(x) e^{-x} = \frac{e^x}{1+e^x}$$

$$\Delta = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -e^{-x} e^x - e^{-x} e^x = -1 - 1 = -2$$

$$\Delta_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{e^x}{1+e^x} & -e^{-x} \end{vmatrix} = 0 - \frac{e^{-x} e^x}{1+e^x} = -\frac{1}{1+e^x}$$

$$\Delta_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{1+e^x} \end{vmatrix} = \frac{e^{2x}}{1+e^x} - 0$$

$$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\frac{1}{1+e^x}}{-2} = \frac{1}{2(1+e^x)}$$

$$1+e^x = t$$

$$e^x dx = dt$$

$$dx = \frac{dt}{e^x} = \frac{dt}{t-1}$$

$$C_1(x) = \int \frac{dx}{2(1+e^x)} \quad \frac{1}{2}\int \frac{dx}{1+e^x}$$

$$\frac{1}{2}\int \frac{\frac{dt}{t-1}}{t} = \int \frac{dt}{t(t-1)} - \int \frac{dt}{t^2-1}$$

$$-\frac{1}{2}\int \frac{dt}{1-t^2} = -\frac{1}{2}\ln\sqrt{\left|\frac{1+t}{1-t}\right|} + C$$

$$\cancel{C_1(x) = -\frac{1}{2}\ln\sqrt{\left|\frac{2+e^x}{2-e^x}\right|}} + D_1 = \frac{1}{2}x - \ln(1+e^x) + D_1$$

$$C_2'(x) = \frac{\Delta_2}{\Delta} = \frac{\frac{e^{2x}}{1+e^x}}{-2} = -\frac{e^{2x}}{2(1+e^x)}$$

$$t-1 \quad \frac{dt}{e^x} \quad e^x dx$$

$$1+e^x = t$$

$$e^x = t-1$$

$$e^x dx = dt$$

$$C_2(x) = -\int \frac{e^{2x}}{2(1+e^x)}dx = -\frac{1}{2}\int \frac{e^{2x}}{1+e^x}dx$$

$$t$$

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$$|1+t|$$

$$= -\frac{1}{2}\int \frac{t-1}{t}dt = -\frac{1}{2}\int dt + \frac{1}{2}\int \frac{dt}{t} = -\frac{1}{2}t + \frac{1}{2}\ln|t| \Rightarrow$$

$$C_2(x) = -\frac{1}{2}(1+e^x) + \frac{1}{2}\ln|1+e^x| = -\frac{1}{2}e^x + \frac{1}{2}\ln(e^x+1) + D_2$$

$$y = \frac{1}{2}x\, e^x + e^x \ln(1+e^x) + D_1 e^x - \frac{1}{2}e^x e^{-x} + \frac{1}{2}e^{-x}\ln(e^x+1) + D_2 e^{-x}$$

$$\frac{e^x}{2x^2}+\frac{1}{2}\left(-\frac{e^x}{x}+\right.$$

$$x^{-3}e^x$$

$u=e^x \quad dv=x^{-3}$

$du=e^x dx \quad v=-\frac{1}{2}x^{-2}$

$$-\frac{e^x}{2x^2}+\frac{1}{2}\int\frac{e^x}{x^2}$$

$u=e^x \quad dv=x^{-2}$

$du=e^x \quad v=-\frac{1}{x}$

$$-\frac{e^x}{x}+\int\frac{e^x}{x}$$

$$x^{-1}e^x$$

$u=e^x \quad dv=\frac{1}{x}$

$du=e^x dx \quad v=\ln x$

$$e^x\ln x-\int e^x\ln x$$

$u=\ln x$

$du=\frac{1}{x}dx$

$dv=e^x dx$

$v=e^x$

$$e^x\ln x-\int\frac{e^x}{x}$$

$$y=C_1(x)e^x+C_2(x)e^{-x}$$

$$y=\left(-\frac{1}{2}\ln\sqrt{\left|\frac{2+e^x}{2-e^x}\right|}+D_1\right)e^x+e^{-x}\left(-\frac{1}{2}(1+e^x)+\frac{1}{2}\ln|1+e^x|+D_2\right)$$

$$=-\frac{1}{2}e^x\ln\sqrt{\left|\frac{2+e^x}{2-e^x}\right|}+e^xD_1-\frac{1}{2}e^{-x}(1+e^x)+\frac{1}{2}e^{-x}(1+e^x)$$
$$+e^{-x}D_2$$

$$y=D_1\cdot e^x+D_2\cdot e^{-x}+\frac{1}{2}e^{-x}\left(\cancel{(1+e^x)}-\cancel{(1+e^x)}\right)-\frac{1}{2}e^x\ln\text{—}$$

$$y=D_1\cdot e^x+D_2\cdot e^{-x}-\frac{1}{2}e^x\ln\sqrt{\left|\frac{2+e^x}{2-e^x}\right|}$$

(7) $y''+y=\operatorname{tg}x \qquad r^2+1=0$

$r^2=-1 \qquad r_2=i \quad r_1=-i$

$$y_{hom}=C_1\cos x+C_2\sin x$$

$$\Delta=\begin{vmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{vmatrix}=\cos^2x+\sin^2x=1$$

$$y_{nh}=C_1(x)\cos x+C_2(x)\sin x$$

$$\Delta_1=\begin{vmatrix}0 & \sin x\\ \operatorname{tg}x & \cos x\end{vmatrix}=0-\sin x\operatorname{tg}x$$

$$\Delta_2=\begin{vmatrix}\cos x & 0\\ -\sin x & \operatorname{tg}x\end{vmatrix}=\cos x\operatorname{tg}x-0=\sin x \qquad \left(\operatorname{tg}x=\frac{\sin x}{\cos x}\right)$$

$$C_1'(x)\cos x+C_2'(x)\sin x=0$$

$$C_1'(x)(-\sin x)+C_2'(x)\cos x=\operatorname{tg}x$$

$$C_1'(x)=\frac{\Delta_1}{\Delta}=\frac{-\sin x\frac{\sin x}{\cos x}}{1}=\frac{-\sin^2x}{\cos x}$$

$$C_1(x)=-\int\frac{\sin^2x}{\cos x}dx=-\int\frac{1-\cos^2x}{\cos x}dx=-\int\frac{dx}{\cos x}+\int\cos x\,dx=-\left(-\ln\left|1-\operatorname{tg}\frac{x}{2}\right|\right)+\sin x+D_1$$

$$C_2'(x)=\frac{\Delta_2}{\Delta}=\sin x \qquad C_2(x)=\int\sin x\,dx=-\cos x+D_2$$

$$y=\left(\sin x-\ln\left|1-\operatorname{tg}\frac{x}{2}\right|+D_1\right)\cos x+(\sin x)(-\cos x+D_2)=\cancel{\sin x}\cos x-\cos x\ln\left|1-\operatorname{tg}\frac{x}{2}\right|$$
$$+D_1\cos x-\sin x\cancel{\cos x}+\sin xD_2$$

$$y=D_1\cos x+D_2\sin x-\left(\ln\left|1-\operatorname{tg}\frac{x}{2}\right|\right)\cos x$$

(9) $y'' - 2y' + y = \frac{x^2+2x+2}{x^3}$

y_{hom} $\quad r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

$r_1 = r_2 = 1$

$y_{hom} = C_1 e^x + C_2 x e^x$

$y_{nch} = C_1(x) e^x + C_2(x) e^x$

$C_1'(x) e^x + C_2'(x) e^x = 0$

$C_1'(x) e^x + C_2'(x) e^x = \frac{x^2+2x+2}{x^3}$

$\Delta = \begin{vmatrix} e^x & e^x \\ e^x & e^x \end{vmatrix} = 0$

$\Delta_1 = \begin{vmatrix} 0 \end{vmatrix}$

(11) $y'' + 4y' + 4y = e^{-2x} \ln x$

$r^2 + 4r + 4 = 0$

$(r+2)^2 = 0 \quad r_1 = r_2 = -2 \quad y_{hom} = C_1 e^{-2x} + C_2 x e^{-2x}$

$y_{nch} = C_1(x) e^{-2x} + C_2(x) x e^{-2x}$

$C_1'(x) e^{-2x} + C_2'(x) x e^{-2x} = 0$

$C_1'(x)(-2e^{-2x}) - 2C_2'(x) e^{-2x} = e^{-2x} \ln x$

$\Delta = \begin{vmatrix} e^{-2x} & x e^{-2x} \\ -2e^{-2x} & -2e^{-2x} \end{vmatrix} = -2e^{-2x}e^{-2x} + 2x e^{-2x} e^{-2x}$

$= 2e^{-4x}(x-2)$

$\Delta_1 = \begin{vmatrix} 0 & x e^{-2x} \\ e^{-2x}\ln x & -2e^{-2x} \end{vmatrix} = 0 - x e^{-4x} \ln x$

$\Delta_2 = \begin{vmatrix} e^{-2x} & 0 \\ -2e^{-2x} & e^{-2x} \ln x \end{vmatrix} = e^{-4x} \ln x - 0$

$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-x \cancel{e^{-4x}} \ln x}{2\cancel{e^{-4x}}(x-2)} = -\frac{x \ln x}{2(x-2)}$

$C_1(x) = -\frac{1}{2} \int \frac{x \ln x}{x-2}\, dx$

$\ln x = t \qquad e^t = x$

$\frac{1}{x} dx = dt$

$dx = x\,dt = e^t dt$

$e^t - 2 = u \quad e^t = u+2 \quad t = \ln(u+2)$

$e^t dt = du$

$\int \frac{e^t \cdot t \cdot e^t}{e^t - 2}\, dt = \frac{t \cdot e^{2t}}{e^t - 2}\, dt$

(15) $\Delta = \begin{vmatrix} e^x & xe^x \\ e^x & e^x \end{vmatrix} = e^{2x} - xe^{2x} = e^{2x}(1-x)$

$\Delta_1 = \begin{vmatrix} 0 & xe^x \\ \frac{x^2+2x+2}{x^3} & e^x \end{vmatrix} = -\cancel{x}e^x \frac{x^2+2x+2}{x^3} = -e^x \frac{x^2+2x+2}{x^2}$

$\Delta_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{x^2+2x+2}{x^3} \end{vmatrix} = e^x \frac{x^2+2x+2}{x^3}$

$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\cancel{e^x}\frac{x^2+2x+2}{x^2}}{e^{2x}(1-x)} = \frac{x^2+2x+2}{x^2e^x(1-x)}$

$C_1(x) = \int \frac{x^2+2x+2}{x^2e^x(1-x)}dx = \int \frac{dx}{e^x(1-x)} + 2\int\frac{dx}{xe^x(1-x)} + 2\int\frac{dx}{x^2e^x(1-x)}$

$\int \frac{dx}{x^2e^x(1-x)} = \int \frac{x^{-2}e^{-x}}{1-x}dx$?

$C_2'(x)e^x(1-x) = -\frac{e^x}{x}$ ... $C_2'(x) = \cancel{\frac{-e^x}{x}}$ ...

(16) $y'' - 2y' + y = \frac{e^x}{x}$

$r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

$r_1 = r_2 = 1$

$y_{hom} = C_1e^x + C_2xe^x$

$y_p = C_1(x)e^x + C_2(x)e^x \cdot x$

$C_1'(x)e^x + C_2'(x)\cdot xe^x = 0$

$C_1'(x)e^x + C_2'(x)e^x = \frac{e^x}{x}$

$\Delta = \begin{vmatrix} e^x & xe^x \\ e^x & e^x \end{vmatrix} = e^{2x} - xe^{2x} = e^{2x}(1-x)$

$\Delta_1 = \begin{vmatrix} 0 & xe^x \\ \frac{e^x}{x} & e^x \end{vmatrix} = 0 - \frac{e^x}{\cancel{x}}\cdot\cancel{x}e^x = -e^{2x}$

$\Delta_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{x} \end{vmatrix} = \frac{e^{2x}}{x}$

$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\cancel{e^{2x}}}{\cancel{e^{2x}}(1-x)} = -\frac{1}{1-x}$

$C_1(x) = \int -\frac{dx}{1-x} = \int \frac{dt}{t} = \ln|t| = \ln|1-x|$

$1-x = t$

$dx = dt$

$1-x = t \quad x = t+1$

$-dx = dt$

$C_2'(x) = \frac{\Delta_2}{\Delta} = \frac{\frac{\cancel{e^{2x}}}{x}}{\cancel{e^{2x}}(1-x)} = \frac{1}{x(1-x)}$

$C_2(x) = \int \frac{dx}{x(1-x)}$

$= \int \frac{dt}{t(t+1)}$

$\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$

$A - Ax + Bx = 1$

$x(\underbrace{B-A}_{0}) + \underbrace{A}_{1} = 1$

$\int\left(\frac{1}{x} + \frac{1}{1-x}\right)dx = \int\frac{dx}{x} + \int\frac{dx}{1-x} = \ln|x| - \ln|1-x|$ $B=1$ $A=1$

$= \ln\left|\frac{x}{1-x}\right|$

$$y = e^x\left(\ln|1-x| + D_1\right) + xe^x\left(\ln|x| - \ln|1-x| + D_2\right)$$
$$= e^x \ln|1-x| + e^x D_1 + xe^x \ln|x| - xe^x \ln|1-x| + xe^x D_2$$

$$y = D_1 e^x + D_2 xe^x + xe^x \ln\left|\frac{x}{1-x}\right| + e^x \ln|1-x|$$

16. $y'' + y = \frac{1}{\sin x}$ $\qquad r^2 + 1 = 0$

$r^2 = -1 \quad r_1 = -i \quad r_2 = i$

$y_{hom} = C_1 \cos x + C_2 \sin x$

$y_{neh} = C_1(x)\cos x + C_2(x)\sin x$

$$\begin{aligned} C_1'(x)\cos x + C_2'(x)\sin x &= 0 \\ -C_1'(x)\sin x + C_2'(x)\cos x &= \frac{1}{\sin x} \end{aligned}$$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \frac{1}{\sin x} & \cos x \end{vmatrix} = 0 - \frac{\sin x}{\sin x} = -1$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\sin x} \end{vmatrix} = \frac{\cos x}{\sin x} = \operatorname{ctg} x$$

$$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-1}{1} = -1$$

$$C_1(x) = \int -1\,dx = -\int dx = -x + D_1$$

$$C_2'(x) = \frac{\Delta_2}{\Delta} = \frac{\cos x}{\sin x}$$

$$C_2(x) = \int \frac{\cos x}{\sin x}\,dx = \ln|\sin x| + D_2$$

$\sin x = t$
$\cos x\,dx = dt$

$$y = (-x + D_1)\cos x + (\ln|\sin x| + D_2)\sin x$$
$$= -x\cos x + D_1\cos x + D_2\sin x + \ln|\sin x| \cdot \sin x$$

17. $y''' + y' = \operatorname{tg} x$ $\qquad r^3 + r = 0$

$r(r^2+1) = 0$

$r_1 = 0 \quad r_2 = -i \quad r_3 = i$

$y_{hom} = C_1 + C_2 \cos x + C_3 \sin x$

$y_{neh} = C_1(x) + C_2(x)\cos x + C_3(x)\sin x$

$$\begin{aligned} C_1'(x)\cdot 1 + C_2'(x)\cdot\cos x + C_3'(x)\cdot\sin x &= 0 \\ 0 + C_2'(x)\sin x + C_3'(x)\cos x &= \operatorname{tg} x \end{aligned}$$

$$\Delta = \begin{vmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \end{vmatrix} \begin{matrix} 1 & \cos x \\ 0 & -\sin x \end{matrix}$$

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⑱ $y'' + y = \frac{1}{\sqrt{\cos 2x}}$ $r^2 + 1 = 0$

$r^2 = -1$ $r_1 = -i$ $r_2 = i$

$y_{hom} = C_1 \cos x + C_2 \sin x$

$y_{part} = C_1(x)\cos x + C_2(x)\sin x$

$C_1'(x)\cos x + C_2'(x)\sin x = 0$

$-C_1'(x)\sin x + C_2'(x)\cos x = \frac{1}{\sqrt{\cos 2x}}$

$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$

$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \frac{1}{\sqrt{\cos 2x}} & \cos x \end{vmatrix} = 0 - \frac{\sin x}{\sqrt{\cos 2x}}$

$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\sqrt{\cos 2x}} \end{vmatrix} = \frac{\cos x}{\sqrt{\cos 2x}}$

$\frac{\cos^2 x}{\cos 2x}$

$\frac{\frac{1+\cos 2x}{2}}{\cos 2x}$

$\frac{1+\cos 2x}{2\cos 2x}$

$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\sin x}{\sqrt{\cos 2x}}$ $C_2'(x) = \frac{\cos x}{\sqrt{\cos 2x}}$

$C_1(x) = -\int \frac{\sin x}{\sqrt{\cos 2x}}\,dx$ $\frac{\sin x}{\sqrt{\cos^2 x - \sin^2 x}}$ $\frac{\cos^2 x - \sin^2 x}{\sin x}$ $\sqrt{2\cos^2 x - 1}$ $\int \frac{1}{2\cos 2x}\,dx$

$\int \frac{\sin^2 x}{\cos 2x}\,dx$ $\frac{1 - \frac{1-\cos 2x}{2}}{\cos 2x}$ $\int \frac{1-\cos 2x}{2\cos 2x}\,dx$ $\frac{dt}{\sqrt{2t^2-1}}$ $\int \frac{\cos 2x}{2\cos 2x}$

$\frac{dt}{\sqrt{2}\sqrt{t^2 - \frac{1}{2}}}$ $\frac{1}{2}\int \frac{dx}{\cos 2x}$

$\frac{1}{2}\int \frac{dx}{\cos 2x} + \frac{1}{2}\int dx$ $\frac{1}{2}\left(-\ln\left|1 - tg\frac{u}{2}\right|\right.$

$\ln$ $-\frac{1}{2}\ln|1 - tg x|$

$C_1(x) = -\frac{1}{2}\left(-\ln|1 - tg x|\right) + \frac{1}{2}x + D_1$

$C_2(x) = -\frac{1}{2}\ln|1 - tg x| + \frac{1}{2}x + D_2$

$y = \left(\frac{1}{2}x + \frac{1}{2}\ln|1 - tg x| + D_1\right)\cos x + \left(\frac{1}{2}x - \frac{1}{2}\ln|1 - tg x| + D_2\right)\sin x$

$D_1\cos x + D_2\sin x + \frac{1}{2}x\cos x + \frac{1}{2}\cos x \ln|1 - tg x| + \frac{1}{2}x\sin x - \frac{1}{2}\ln|1 - tg x|\sin x$

(17) $y''' + y' = tg\,x$ $\quad r^3 + r = 0 \quad r(r^2+1) = 0 \quad r_1 = 0 \quad r_2 = -i \quad r_3 = i$

$y_{hom} = C_1 + C_2 \cos x + C_3 \sin x$

$y_{neh} = C_1(x) + C_2(x)\cos x + C_3(x)\cdot\sin x$

$C_1'(x) + C_2'(x)\cos x + C_3'(x)\sin x = 0$

$0 + C_2'(x)(-\sin x) + C_3'(x)\cos x = 0$

$0 + C_2'(x)(-\cos x) + C_3'(x)(-\sin x) = tg\,x$

$$\Delta = \begin{vmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 0 & -\cos x & -\sin x \end{vmatrix} \begin{matrix} 1 & \cos x \\ 0 & -\sin x \\ 0 & -\cos x \end{matrix} = (0 - \cos^2 x + 0) - (\sin^2 x + 0 + 0)$$

$- \cos^2 x - \sin^2 x$

$-(\cos^2 x + \sin^2 x) = -1$

$$\Delta_1 = \begin{vmatrix} 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ tg\,x & -\cos x & -\sin x \end{vmatrix} \begin{matrix} 0 & \cos x \\ 0 & -\sin x \\ tg\,x & -\cos x \end{matrix} = (tg\,x(-\sin^2 x) + 0 + 0) - (0 + tg\,x \cos^2 x + 0)$$

$tg\,x(-\sin^2 x - \cos^2 x)$

$-tg\,x(\sin^2 x + \cos^2 x) = -tg\,x$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & \sin x \\ 0 & 0 & \cos x \\ 0 & tg\,x & -\sin x \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 0 \\ 0 & tg\,x \end{matrix} = (0 + \cos x \cdot tg\,x + 0) - (0 + 0 + 0)$$

$tg\,x \cdot \cos x = \underline{\sin x}$

$$\Delta_3 = \begin{vmatrix} 1 & \cos x & 0 \\ 0 & -\sin x & 0 \\ 0 & -\cos x & tg\,x \end{vmatrix} \begin{matrix} 1 & \cos x \\ 0 & -\sin x \\ 0 & -\cos x \end{matrix} = ((-\sin x)\,tg\,x + 0 + 0) - (0 + 0 + 0)$$

$-tg\,x \cdot \sin x = \dfrac{-\sin^2 x}{\cos x}$

$\cos x = t$
$-\sin x\,dx = dt$

$C_1'(x) = \dfrac{\Delta_1}{\Delta} = \dfrac{-tg\,x}{-1} = tg\,x = \dfrac{\sin x}{\cos x} \qquad C_1(x) = \displaystyle\int \frac{\sin x}{\cos x}\,dx = \underline{-\ln|\cos x| + D_1}$

$C_2'(x) = \dfrac{\Delta_2}{\Delta} = \dfrac{\sin x}{-1} = -\sin x \qquad C_2(x) = \displaystyle\int -\sin x\,dx = -\int \sin x\,dx = \underline{\cos x + D_2}$

$C_3'(x) = \dfrac{\Delta_3}{\Delta} = \dfrac{\sin^2 x}{\cos x} \qquad C_3(x) = \displaystyle\int \frac{\sin^2 x}{\cos x}\,dx = \int \frac{1-\cos^2 x}{\cos x}\,dx = \int \frac{dx}{\cos x} - \int \cos x\,dx$

$= -\ln\left|1 - tg\frac{x}{2}\right| - \sin x + D_3$

$$y = \left(-\ln|\cos x| + D_1\right) + \left(\cos x + D_2\right)\cdot\cos x + \left(-\ln\left|1-\operatorname{tg}\frac{x}{2}\right| - \sin x + D_3\right)\cdot \sin x$$

$$y = D_1 + D_2\cdot\cos x + D_3 \sin x \underline{- \ln|\cos x| + \cos^2 x - \ln\left|1-\operatorname{tg}\frac{x}{2}\right|\cdot\sin x - \sin^2 x}$$

(19) $y'' + 5y' + 6y = \dfrac{1}{1+e^{2x}}$ $\qquad r^2+5r+6=0$

$r_{1,2} = \dfrac{-5\pm 1}{2}$ $\quad \underline{r_1=-2}$ $\quad \underline{r_2=-3}$

$y_{hom} = C_1 e^{-2x} + C_2 e^{-3x}$

$y_{neh} = C_1(x)e^{-2x} + C_2(x)e^{-3x}$

$$C_1'(x)e^{-2x} + C_2(x)e^{-3x} = 0$$

$$C_1'(x)(-2e^{-2x}) + C_2(x)(-3e^{-3x}) = \frac{1}{1+e^{2x}}$$

$$\Delta = \begin{vmatrix} e^{-2x} & e^{-3x} \\ -2e^{-2x} & -3e^{-3x}\end{vmatrix} = -3e^{-3x}\cdot e^{-2x} + 2e^{-2x}\cdot e^{-3x}$$

$$-3e^{-5x} + 2e^{-5x} = \underline{-e^{-5x}}$$

$$\Delta_1 = \begin{vmatrix} 0 & e^{-3x} \\ \frac{1}{1+e^{2x}} & -3e^{-3x}\end{vmatrix} = 0 - \frac{e^{-3x}}{1+e^{2x}} \qquad \Delta_2 = \begin{vmatrix} e^{-2x} & 0 \\ -2e^{-2x} & \frac{1}{1+e^{2x}}\end{vmatrix} = \frac{e^{-2x}}{1+e^{2x}} - 0$$

$$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{\frac{-e^{-3x}}{1+e^{2x}}}{-e^{-5x}} = \frac{e^{2x}}{1+e^{2x}}$$

$$C_1(x) = \int \frac{e^{2x}}{1+e^{2x}}\,dx = \frac{1}{2}\int\frac{dt}{t} = \underline{\underline{\frac{1}{2}\ln|1+e^{2x}| + D_1}}$$

$1+e^{2x} = t$

$2e^{2x}dx = dt$

$$C_2'(x) = \frac{\Delta_2}{\Delta} = \frac{\frac{e^{-2x}}{1+e^{2x}}}{-e^{-5x}} = -\frac{e^{3x}}{1+e^{2x}}$$

$1+e^{2x}=t$ $\qquad e^x = t$

$1+e^{2x}=t^2$

$e^x dx = dt$

$\cancel{2}e^{2x}dx = \cancel{2}t\,dt$

$$C_2(x) = -\int\frac{e^{3x}}{1+e^{2x}}\,dx \qquad \frac{e^x\cdot e^{2x}}{1+e^{2x}}\,dx \qquad \int\frac{t^2\,dt}{1+t^2}$$

$$\int\frac{t^2+1-1}{t^2+1}\,dt = \int dt - \int\frac{dt}{1+t^2}$$

$$t - \operatorname{arctg} t \to \underline{\underline{e^x - \operatorname{arctg} e^x}}$$

$$y = \left(\frac{1}{2}\ln|1+e^{2x}| + D_1\right)e^{-2x} + \left(e^x - \operatorname{arctg} e^x + D_2\right)\cdot e^{-3x}$$

(25) $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ $\qquad r^2 - 6r + 9 = 0$

$(r-3)^2 = 0 \quad r_1 = r_2 = 3$

$y_{hom} = C_1 e^{3x} + C_2 x e^{3x}$

$y_{neh} = C_1(x) e^{3x} + C_2(x)\, x e^{3x}$

$C_1'(x) e^{3x} + C_2'(x)\, x e^{3x} = 0$

$C_1'(x) \cdot 3e^{3x} + C_2'(x)\, 3e^{3x} = \frac{e^{3x}}{x^2}$

$\Delta = \begin{vmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & 3e^{3x} \end{vmatrix} = 3e^{6x} - 3x e^{6x} = 3e^{6x}(1-x)$

$\Delta_1 \begin{vmatrix} 0 & xe^{3x} \\ \frac{e^{3x}}{x^2} & 3e^{3x} \end{vmatrix} = -\cancel{x} e^{3x} \frac{e^{3x}}{\cancel{x^2}} = \frac{-e^{6x}}{x}$

$\Delta_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & \frac{e^{3x}}{x^2} \end{vmatrix} = \frac{e^{6x}}{x^2}$

$C_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\frac{\cancel{e^{6x}}}{x}}{3\cancel{e^{6x}}(1-x)} = -\frac{1}{3(1-x)}$

$C_1(x) = -\frac{1}{3}\int \frac{dx}{1-x} = \frac{1}{3}\ln|1-x| + D_1$

$C_2'(x) = \frac{\Delta_2}{\Delta} = \frac{\frac{\cancel{e^{6x}}}{x^2}}{3\cancel{e^{6x}}(1-x)} = \frac{1}{3x^2(1-x)}$

$C_2(x) = \frac{1}{3}\int \frac{dx}{\cancel{3}x^2(1-x)}$

$\frac{1}{x^2(1-x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1-x}$

$Ax(1-x) + B(1-x) + Cx^2 = 1$

$Ax - Ax^2 + B - Bx + Cx^2 = 1$

$x^2(C-A) + x(A-B) + B = 1$ $\qquad \left(\ln\left|\frac{x}{1-x}\right| - \frac{1}{x} + D_2\right)$

$C = 1 \quad A = 1 \quad B = 1$

$\int \frac{1}{x}dx + \int \frac{1}{x^2}dx + \int \frac{1}{1-x}dx = \ln|x| - \frac{1}{x} - \ln|1-x| + D_2$

$y = e^{3x}\left(\frac{1}{3}\ln|1-x| + D_1\right) + xe^{3x}\left(\ln\left|\frac{x}{1-x}\right| - \frac{1}{x} + D_2\right)$

5.5. Luivilova formula

(4) $x(1-x)y'' + (2x^2-1)y' + (2-4x)y = 0 \,/ : x(1-x)$ $y_1 = ae^{2x}$

$$y_2(x) = y_1(x)\int \frac{e^{-\int p(x)dx}}{y_1^2(x)}dx \qquad (a^x)^2$$

$y_1 = ae^{2x}$ $y_1' = 2ae^{2x}$ $y_1'' = 4ae^{2x}$

d.j.

$$x(1-x)\cdot 4a\cancel{e^{2x}} + (2x^2-1)\cdot 2a\cancel{e^{2x}} + (2-4x)\cdot a\cancel{e^{2x}} = 0$$

$$4ax(1-x) + 2a(2x^2-1) + a(2-4x)$$

$$\cancel{4ax} - \cancel{4ax^2} + \cancel{4ax^2} - \cancel{2a} + \cancel{2a} - \cancel{4ax} = 0$$

$\boxed{0=0}$ $\forall a \in R$

$\int(2x^2-1)dx$

$2\int x^2dx - \int dx$

$= 2\frac{x^3}{3} + x$

npr. $a=1$

$y_1 = e^{2x}$ $y_2(x) = e^{2x}\int \frac{e^{-\int \frac{2x^2-1}{x-x^2}dx}}{(e^{2x})^2}dx$

$$y'' + \frac{2x^2-1}{x(1-x)}y' + \frac{2-4x}{x(1-x)}y = 0$$

$-\int\left(\frac{2x^2-1}{-(x^2-x)}\right)dx$

$$\int \frac{2x^2-1}{x(1-x)}dx = -\int\frac{2x^2}{x-x^2}dx - \int\frac{1}{x-x^2}dx = -\int$$

$-2\int \frac{x^2-x+x}{x^2-x}$ $-(-2(x-\ln|1-x|))$

$2x - 2\ln|1-x|$

$\frac{x^2-x+x}{x^2-x}$ $\int dx + \int\frac{x}{x(1-x)}$

$$-2\left[\int dx + \int\frac{dx}{1-x}\right] = -2(x - \ln|1-x|)$$

$= -2\ln|1-x| + 2x$

$$y_2 = e^{2x}\int\frac{e^{2(x-\ln|1-x|)}}{e^{4x}}dx \qquad \ln(1-x)^{-2}$$

$$y_2 = e^{2x}\int\frac{\cancel{e^{2x}}\cdot e^{-2\ln|1-x|}}{e^{\cancel{4x}\,2x}}dx$$

$$y_2 = e^{2x}\int\frac{dx}{e^{2x}(1-x)^2}$$

$y(x) = y_1 \cdot z(x)$ $\qquad y = e^{2x} \cdot z(x)$

$$y' = 2e^{2x} z(x) + e^{2x} \cdot z'(x)$$

$$y'' = 4e^{2x} z(x) + 2e^{2x} z'(x) + 2e^{2x} z'(x) + e^{2x} z''(x)$$

$$4e^{2x}(zx) + z'(x)(4e^{2x}) + e^{2x} \cdot z''(x)$$

$$x(1-x) \cdot e^{2x}\left(4z(x) + 4z'(x) + z''(x)\right) + (2x^2-1) \cdot e^{2x}\left(2z(x) + z'(x)\right) +$$
$$+ (2-4x) \cdot e^{2x} z(x) = 0$$

$$z''(x) \cdot \left[x(1-x) \cdot e^{2x}\right] + z'(x)\left[4e^{2x} \cdot x(1-x) + e^{2x}(2x^2-1)\right]$$
$$+ z(x)\left[4e^{2x} \cdot x(1-x) + 2e^{2x}(2x^2-1) + e^{2x}(2-4x)\right] = 0$$

$$z''(x)\left[e^{2x} \cdot x(1-x)\right] + z'(x)\left[e^{2x}\left(4x(1-x) + (2x^2-1)\right)\right] + z(x)\left[e^{2x}\left(4x(1-x) + 2(2x^2-1) + (2-4x)\right)\right] = 0$$

$4x - 4x^2 + 2x^2 - 1$ $\qquad 4x - 4x^2 + 4x^2 - 2 + 2 - 4x$

$$z''(x)\left(e^{2x} \cdot x(1-x)\right) + z'(x)\left[e^{2x}\left(4x - 2x^2 - 1\right)\right] + z(x)\left[e^{2x} \cdot 0\right] = 0$$

$$z'' \cdot e^{2x} \cdot x(1-x) + z' e^{2x}\left(-2x^2 + 4x - 1\right) = 0$$

$$z'' \cdot \cancel{e^{2x}} \cdot x(1-x) = -z' \cancel{e^{2x}}\left(-2x^2 + 4x - 1\right)$$

$$\frac{dz'}{dx} x(1-x) = (2x^2 - 4x + 1) z'$$

$$\frac{dz'}{z'} = \frac{2x^2 - 4x + 1}{-x^2 + x} dx \Big/ \int$$

$x - x^2$
$-(x^2 - x)$

$$\int \frac{dz'}{z'} = \int \underbrace{\frac{2x^2 - 4x + 1}{x^2 - x}} dx \qquad \int \frac{1}{x(x-1)}$$

$$\int \frac{2x^2}{x^2 - x} - \int \frac{4x}{x^2 - x} + \int \frac{1}{x^2 - x}$$

$$2\int \frac{x^2 - x + x}{x^2 - x} - 4\int \frac{x}{x^2 - x} + \int \frac{1}{x^2 - x}$$

$$\frac{A}{x} + \frac{B}{x-1} = \frac{1}{x(x-1)}$$

$$2\int dx + 2\int \frac{x}{x^2 - x} - 4\int \frac{x}{x^2 - x} + \int \frac{1}{x^2 - x}$$

$Ax - A + Bx = 1$

$x(A + B) - A = 1$

$$2\int dx - 2\int \frac{x}{x^2 - x} + \int \frac{1}{x(x-1)}$$

$-A = 1$

$A = -1$

$B = 1$

$$\int \frac{x}{x^2 - x} dx \qquad \int \frac{\cancel{x}}{\cancel{x}(1-x)} \qquad 2\int dx - 2\int \frac{dx}{1-x} + \int \frac{dx}{x(x-1)}$$

$x - 1 = t$

$dx = dt$

$$-\int \frac{dx}{x-1} \qquad -\int \frac{1}{x} + \int \frac{1}{x-1}$$

$$-2\int \frac{dx}{x-1} \qquad -\ln|x| + \ln|x-1|$$

$$\int \frac{2x^2-4x+1}{x^2-x}dx = 2x + 2\ln|1-x| - \ln|x| + \ln|x-1|$$

$$2x + 2\ln|x-1| + \ln|x-1| - \ln|x|$$

$$2x + 3\ln|x-1| - \ln|x| = 2x + \ln\left|\frac{(x-1)^3}{x}\right|$$

$$\ln|z'| = 2x + \ln\left|\frac{(x-1)^3}{x}\right| + c_1$$

$$\ln_e = x$$

$$z' = e^{2x + \ln\left|\frac{(x-1)^3}{x}\right| + c_1}$$

$$e^{2x} = t$$

$$z' = e^{2x} \cdot e^{\ln\left|\frac{(x-1)^3}{x}\right|} \cdot e^{c_1} \text{ nova } C$$

$$2x = \ln|t| \qquad x-1 = \frac{1}{2}\ln t - 1$$

$$z' = e^{2x} \cdot \frac{(x-1)^3}{x} \cdot c_1$$

$$x = \frac{1}{2}\ln t$$

$$dx = \frac{1}{2t}dt$$

$$z = \int \frac{e^{2x}(x-1)^3}{x} c_1\, dx + c_2 = \int \frac{\cancel{t} \cdot \left(\frac{\ln t}{2} - 1\right)}{\frac{\ln t}{2}} \cdot \frac{1}{2\cancel{t}} dt$$

$$\frac{1}{2}\int \frac{\ln t - \frac{1}{2}}{\ln t} dt \qquad \frac{\cancel{t}\left(\frac{\ln t}{2} - 1\right)}{\ln t} \cdot \frac{1}{\cancel{t}} dt$$

$$\frac{1}{2}\int \frac{\ln t}{\ln t} dt - \frac{1}{2}\int \frac{\frac{1}{2}dt}{\ln t}$$

$$\ln t = u$$

$$\frac{1}{2}\int dt - \frac{1}{4}\int \frac{dt}{\ln t}$$

$$t = e^{u}$$

$$dt = e^{u} du$$

$$\int \frac{e^u du}{u} \qquad \int u^{-1} e^u du$$

$$u = e^u \qquad dv = \frac{1}{u} = $$

$$\int \frac{e^u}{u} du = e^u \ln u - e^u \ln u \qquad du = e^u du \qquad v = \ln u$$

$$e^u \ln u - \int \ln u \cdot e^u du$$

$$u = \ln u \qquad dv = e^u$$

$$du = \frac{1}{u} du \qquad v = e^u$$

$$e^u \ln u - \int \frac{e^u}{u} du$$

4.1.

26. $y' = xy^2 \operatorname{ch} x$

$\frac{dy}{dx} = xy^2 \operatorname{ch} x$

$\frac{dy}{y^2} = dx \cdot x \operatorname{ch} x$

$\int \frac{dy}{y^2} = \int x \operatorname{ch} x \, dx$

$-\frac{1}{y} = x \operatorname{sh} x - \operatorname{ch} x$

$y = \frac{-1}{x \operatorname{sh} x - \operatorname{ch} x}$

$\int \operatorname{ch} x = \operatorname{sh} x$

$\int \operatorname{sh} x = \operatorname{ch} x$

$u = x \qquad dv = \operatorname{ch} x$

$du = dx \qquad v = \operatorname{sh} x$

$\frac{1}{e^x + e^{-x}}$

---

21. $xyy' - y^2 = (x+y)^2 e^{-\frac{y}{x}} \quad / : xy$

$y xy' = \frac{(x+y)^2 e^{-\frac{y}{x}}}{} + y^2$

$y' = \frac{(x+y)^2 e^{-\frac{y}{x}} + y^2 \cdot xy}{xy}$

$y' - \frac{y}{x} = \frac{(x+y)^2}{xy} e^{-\frac{y}{x}}$

$y' = \frac{y}{x} + e^{-\frac{y}{x}} \cdot \frac{x^2 + 2xy + y^2}{xy} = \frac{x^2(1 + 2\frac{y}{x} + \frac{y^2}{x^2})}{xy}$

$u + e^{-u} \cdot \frac{(1+u)^2}{u}$

$u = \frac{y}{x} \qquad y = ux$

$\frac{(x^2 + 2xy + y^2) e^{-u} + y^2 xy}{xy}$

$\frac{x^2(1 + 2\frac{y}{x} + \frac{y^2}{x^2}) e^{-u}}{xy}$

$\frac{x}{y}\left(1 + 2\frac{y}{x} + \frac{y^2}{x^2}\right) e^{-u} + y^2$

$\frac{1}{u}\left(1 + 2u + u^2\right) e^{-u}$

$u + u'x = \frac{1 + 2u + u^2}{u \cdot e^u} + u^2 x^2$

$\frac{1}{u}(1 + 2u + u^2) e^{-u} = \frac{1 + 2u + u^2}{u \cdot e^u} + u$

$u'x = \frac{1 + 2u + u^2}{ue^u} + \cancel{u} - \cancel{u}$

$u'x = \frac{u^2 + 2u + 1}{u \cdot e^u}$

$\frac{du}{dx} x = \frac{u^2 + 2u + 1}{u \cdot e^u} = \frac{(u+1)^2}{u \cdot e^u}$

$\frac{u \cdot e^u}{(u+1)^2} du = \frac{dx}{x} \quad / \int$

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$$u' \cdot x = \frac{1+2u+u^2+u^3x^2e^u-u^2e^u}{u \cdot e^u}$$

$$\frac{du}{dx}x = u^3x^2 + \frac{1+2u+u^2}{u \cdot e^u} - u$$

$$\int \frac{u \cdot e^u}{\underbrace{u^2+2u+1}_{(u+1)^2}} du \qquad \frac{u \cdot e^u}{(1+u)^2}$$

$e^u = t$

$u = \ln(t)$ $\quad du = \frac{1}{t} dt$

$$\frac{t \cdot \ln t}{(\ln t+1)^2} \frac{1}{t} dt \qquad \frac{\ln t}{(\ln t+1)^2} dt$$

$\ln t + 1 - \ln|\ln t + 1| = \ln c$

$(u+1) - \ln|u+1|$

$u+1 - \ln|u+1| = \ln|cx|$

$\ln t + 1 = X$

$\frac{1}{t} dt = dx$ $\quad dt = x dx$

$\ln t = x - 1$

$$\frac{x-1}{x^2} \cdot x dx$$

$$\frac{x-1}{x} dx$$

$$\int dx - \int \frac{dx}{x}$$

$x - \ln|x|$

$\ln t + 1 - \ln|\ln t + 1|$

(20) $y\,dx + (2x - y^2)\,dy = 0$

$y\,dx = -dy(2x - y^2)$

$$\frac{dy}{dx} = \frac{-y}{2x-y^2} \quad /^{-1}$$

$\cancel{xdn = -\frac{2x-y^2}{y}}$

$x = v + a \quad y = z + b$

$-y = -z - b$

$2x - y^2 = 2v + 2a - z^2 - 2zb - b^2$

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20 $y\,dx + (2x - y^2)\,dy = 0$

$$\frac{dy}{dx} = \frac{-y}{2x - y^2} \quad /^{-1}$$

$$\frac{dx}{dy} = \frac{2x - y^2}{-y} \qquad \cancel{-y \cdot x' = 2x - y^2}$$

$$x' = -2\frac{x}{y} + y$$

$$x' + \frac{2}{y}x = y$$

$$x(y) = e^{-\int \frac{2}{y}dy}\left[c + \int y\, e^{\int \frac{2}{y}dy}\,dy\right]$$

$$x(y) = e^{-2\ln|y|}\left[c + \int y\, e^{2\ln|y|}\,dy\right]$$

$$x(y) = e^{\ln|y|^{-2}}\left[c + \int y\, e^{\ln|y|^2}\,dy\right] = \frac{1}{y^2}\left[c + \int y \cdot y^2\,dy\right]$$

$$= \frac{1}{y^2}\left[c + \frac{1}{3}y^3\right]$$

$$= \frac{1}{y^2}C + \frac{1}{3}y^2 + D$$

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19. $y\,dx - (y^2\sqrt{x} + 4x)\,dy = 0$

$$y\,dx = y^2\sqrt{x} + 4x\ dy$$

$$\frac{dy}{dx} = \frac{y}{y^2\sqrt{x} + 4x}$$

$$y' = \frac{y}{y^2\sqrt{x} + 4x} \quad /^{-1} \qquad x = z^2$$

$$x' = 2z \cdot z'$$

$$x' = \frac{y^2\sqrt{x} + 4x}{y} = y\sqrt{x} + \frac{4x}{y}$$

$$x' - \frac{4}{y}x = y\,x^{\frac{1}{2}} \qquad \alpha = \frac{1}{2} \qquad z = x^{1-\alpha} = x^{\frac{1}{2}} = \sqrt{x}$$

$$x = z^2$$

$\underline{d)}\quad 2z\cdot z' - \frac{4}{y}\circ z^2 = y\cdot z \;/ : 2z$

$z' - \frac{z}{y} = \frac{1}{2}y$

$p(y) = -\frac{1}{y} \qquad g(y) = \frac{1}{2}y$

$z(y) = e^{-\int -\frac{1}{y}dy}\left[c + \int \frac{1}{2}y\, e^{\int -\frac{1}{y}dy}dy\right] = e^{\ln y}\left[c + \frac{1}{2}\int y\, e^{-\ln y}\, dy\right]$

$= y\left[c + \frac{1}{2}\int \cancel{y}\frac{1}{\cancel{y}}dy\right] = y\left[c + \frac{1}{2}y\right] = \underline{\underline{yc + \frac{1}{2}y^2}}$

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15) $\underbrace{(y^3 - 2xy)}_{P(x)}dx + (3xy^2 - x^2 + 3y^2)\,dy = 0 \qquad P'_y = Q'_x$

$3y^2 - 2x = 3y^2 - 2x$ ✓

$P(x) = u'_x = y^3 - 2xy$

$u = \int (y^3 - 2xy)\,d(x) + C(y) = \qquad x$